

# Estimation of thermal conduction loads for structural supports of cryogenic spacecraft assemblies

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18 February 2004

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## Abstract

Early in the design process of cryogenic space systems there is a critical need for means of estimating the parasitic conduction loads associated with structural supports. In a mature design, the conduction loads can be computed with good accuracy based on the design details. However, for a generic trade study early in the design process, what is desired is a generic relationship (for typical launch loading conditions) between overall support conductance, the  $\Delta T$  involved, and the supported mass. This work derives such a universal relationship by examining a variety of flight-proven designs for cryogenic structural supports and then normalizing the data given the known relationships between material conductivity and temperature, between launch acceleration level and assembly mass, between launch acceleration loads and stresses and required support-member cross-sections, and between support-member cross-section and conductive load.

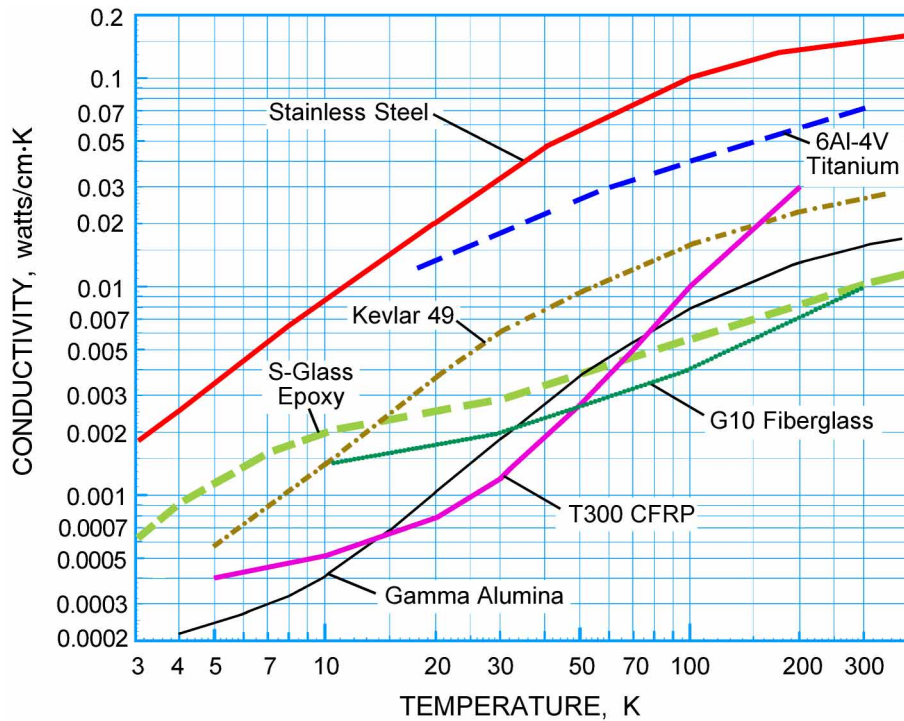
*Keywords:* Thermal conduction loads; Spacecraft cryogenic supports; Launch loads; Conductivity

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## Introduction

For structural support conduction there are three key issues: 1) achieving a high strength and low thermal conductivity structural support design using low conductivity, high strength materials such as glass, fiberglass, Kevlar and Gamma Alumina, 2) supporting the assembly from an intermediate-temperature support to reduce the  $\Delta T$  across the support structure, and 3) minimizing the mass of the assembly to reduce the structural loads that the supports must carry. In a mature design, the conduction loads can be computed with good accuracy based on the design details. However, for a generic trade study early in the design process, what is desired is a generic relationship (for typical launch loading conditions and material stress utilization) between overall support conductance, the  $\Delta T$  involved, and the supported mass.

One means of deriving such a relationship is to examine a variety of flight-proven designs for cryogenic structural supports and to normalize the data given the known relationships between material conductivity and temperature, between launch acceleration level and assembly mass, between launch acceleration loads and required support-member cross-sections, and between support-member cross-section and conductive load. Each of these dependencies is first discussed independently in the sections that follow, then the relationships are combined to provide a universal relationship for conductive load estimation.



**Figure 1.** Conductivity versus temperature for representative cryogenic structural materials.

### Conductive load dependences

It is useful to first recall the generic parameters governing thermal conduction loads associated with heat transfer down structural supports that have a differential temperature ( $\Delta T$ ) along their length. For simple axial members we have:

$$Q = \kappa \Delta T (A / L) \quad (1)$$

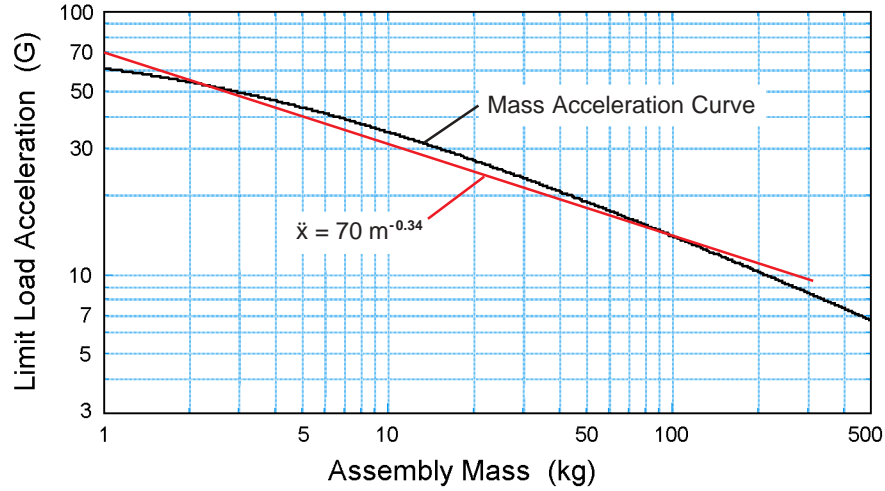
where

- $Q$  = Conducted heat (watts)
- $\kappa$  = Material conductivity (watts/cm·K)
- $\Delta T$  = Differential temperature along member length, K
- $A$  = Structural member cross-sectional area (cm<sup>2</sup>)
- $L$  = Structural member length (cm)

Equation (1) is exact for the case where the material conductivity ( $\kappa$ ) and area ( $A$ ) are constant down the length of the member. For more complex cases,  $Q$  must be determined by integrating the thermal properties over the length of the member using temperature dependent values of ( $\kappa$ ) and variable member cross-sectional areas.

### Temperature dependence of conductivity

Although material conductivity ( $\kappa$ ) is relatively constant near room temperature, it varies dramatically with temperature over the broad range of cryogenic temperatures. Temperature dependence of conductivity is thus important to take into account when estimating the conductance loads into cryogenic assemblies. Conductivity varies dramatically from material to material, both in absolute level and in its relative temperature dependence. Figure 1 presents plots for the temperature dependence of a number of representative materials including Kevlar, stainless steel, 6-4 titanium, epoxy S-glass bands, and gamma alumina. Note the significant difference between these materials.



**Figure 2.** Representative mass acceleration curve for cryogenic assemblies.

### *Dependence of member conductance on launch acceleration loads*

As seen in Eq. (1), the second key issue in estimating member conductance is estimating the overall area/length ( $A/L$ ) ratio required for the support system. This process normally requires a detailed structural analysis based on known member geometries, the supported assembly mass, and the input launch acceleration level. However, for preliminary estimation purposes what is needed is a means of scaling previously developed designs for different supported masses and their associated launch acceleration levels.

To develop this scaling it is first noted that for simple tension-compression type members, which are a common form of cryogenic support, the required cross-sectional area increases directly proportional to the accelerating force and inversely proportional to the mechanical stress reached in the material when the force is applied, i.e.  $A = F/\bar{\sigma}$ . Also, from Newton's laws, the force is proportional to the assembly mass times its acceleration. Thus

$$A \propto m \ddot{x} / \bar{\sigma} \quad (2)$$

where

- $A$  = Total support cross-sectional area ( $\text{cm}^2$ )
- $m$  = Supported assembly's mass (kg)
- $\ddot{x}$  = Assembly peak acceleration during launch (Gs)
- $\bar{\sigma}$  = Average stress achieved in structural member during peak acceleration (MPa)

Next, we can obtain a relationship between an assembly's mass and its acceleration level by appealing to a representative Mass Acceleration Curve (MAC), as shown in Fig. 2. The MAC is a NASA-approved reliability practice for estimation of the maximum expected launch loading acceleration of appendages with masses less than 500 kg, regardless of location, orientation, or natural frequency.<sup>1</sup> Such curves are derived for each launch vehicle based on analytical and flight data, and include the effects of both low-frequency launch vehicle acceleration transients and mechanically transmitted random vibration derived from acoustic loading of the vehicle and payload surfaces. For very low-mass items (less than 1 kg) the levels for 1 kg are used. Although the curves vary slightly for different launch vehicles, the differences are generally small, and one can approximate the dependence between mass and acceleration by the straight line in Fig. 2, given by

$$\ddot{x} \propto m^{-0.34} \quad (3)$$

Combining Eq. (3) with Eq. (2) gives:

$$A \propto m \times m^{-0.34}/\bar{\sigma} = m^{0.66}/\bar{\sigma} \quad (4)$$

Thus we get:

$$(A/L)_2 = (A/L)_1(\bar{\sigma}_1/\bar{\sigma}_2)(m_2/m_1)^{0.66} \quad (5)$$

where

- $(A/L)_1$  = A/L determined for assembly 1
- $(A/L)_2$  = A/L estimate for assembly 2
- $m_1$  = mass of assembly 1
- $m_2$  = mass of assembly 2
- $\bar{\sigma}_1$  = Average stress achieved in members of assembly 1 (MPa)
- $\bar{\sigma}_2$  = Average stress achieved in members of assembly 2 (MPa)

### *Overall scaling equation*

Combining Eqs.(1) and (5) allows measured cryogenic loads to be scaled for both different temperatures, different average stresses, and different supported masses. This composite relationship is thus

$$Q_2 = Q_1 (\kappa_2/\kappa_1)(\bar{\sigma}_1/\bar{\sigma}_2)(m_2/m_1)^{0.66}(\Delta T_2/\Delta T_1) \quad (6)$$

where

- $Q_1$  = Conductive load measured for assembly 1
- $Q_2$  = Conductive load estimated for assembly 2
- $\kappa_1$  = Average conductivity of support material in temperature range  $\Delta T_1$
- $\kappa_2$  = Average conductivity of support material in temperature range  $\Delta T_2$
- $m_1$  = Mass of assembly 1
- $m_2$  = Mass of assembly 2
- $\Delta T_1$  = Differential temperature for assembly 1
- $\Delta T_2$  = Differential temperature for assembly 2
- $\bar{\sigma}_1$  = Average stress achieved in members of assembly 1 (MPa)
- $\bar{\sigma}_2$  = Average stress achieved in members of assembly 2 (MPa)

When the material geometry is known, but the thermal load is not measured, the following alternate form of Eq. (6) is useful

$$Q_2 = \kappa_2 (\bar{\sigma}_1/\bar{\sigma}_2)(m_2/m_1)^{0.66} \Delta T_2 (A_1/L_1) \quad (7)$$

In the above expressions the average achieved stress ( $\bar{\sigma}$ ) is not the material's capability (yield or fatigue limit stress), but rather the actual average stress achieved in the material during launch loading. Thus, it reflects the actual design practices used as limited by such things as geometric configuration, required factors of safety, considerations of buckling, and limitations of the material's ultimate or yield strength.

For a truss or band structure with pure axially loaded members, the average achieved stress may closely approach the material's stress capability. In contrast, for a constant cross-section cantilever structure, much of the material may be at a low stress compared to that at the cantilever's root, and the average achieved stress will be well below the material's capability. The same is true for members in bending where the material near the neutral axis will be at a much reduced stress level. Such designs with a low average achieved stress will have a higher average cross-sectional area and a higher structural conduction than comparable-weight structures that achieve a high stress utilization.

**Table 1.** Derivation of structural support conduction scaling constant  $\bar{A}$ .

Instrument	Suspend. Mass (kg)	Cryogenic Support Type	Standoff Temperature $\Delta T_o$ (K)	Average Conductivity $\kappa_o$ (W/cm·K)	Conduction Load $Q_o$ (W)	Conduction Constant $\bar{A}$
AIRS <sup>[2]</sup>	0.597	hermetic glass tube	160-55=105	0.0075	0.160	0.285
MICAS <sup>[3]</sup>	0.570	S-glass tension bands	140-100=40	0.0063	0.048	0.276
Spire <sup>[4]</sup>	0.450	Kevlar tension bands	2-0.1=1.9	0.00002	(A/L=0.15 cm)	0.254
TES <sup>[5]</sup>	1.000	Z-fold S-glass tube	180-65=115	0.006	0.163	0.236
Thematic Mapper <sup>[6]</sup>	0.6	S-glass tension bands	140-80=60	0.006	0.010	0.039
Hessi <sup>[7]</sup>	26	S-glass tension bands	300-80=220	0.0075	(A/L=0.19 cm)	0.022
MAP <sup>[8]</sup>	50	Gamma-Alumina Struts	287-100=187	0.010	0.555	0.022
DoD Instrument	43	Gamma-Alumina Struts	293-75=218	0.010	(A/L=0.212 cm)	0.018

### Rule-of-thumb estimation based on past designs

Given Eqs. (6) and (7) we can now gather together a number of historical cryogenic support designs with proven launch vibration performance and derive a general relationship for predicting conduction loads. Thus

$$Q \approx \bar{A} \kappa m^{0.66} \Delta T \quad (8)$$

where

- $Q$  = Conductive load being estimated (watts)
- $\bar{A}$  = Empirical scaling factor =  $Q_o / (\kappa_o m_o^{0.66} \Delta T_o) = (A_o / L_o) / m_o^{0.66}$
- $\kappa$  = Average conductivity of support material in temperature range  $\Delta T$  (watts/cm·K)
- $m$  = Mass of supported assembly (kg)
- $\Delta T$  = Differential temperature across support structure (K)
- $A/L$  = Member area/length ratio (cm)

Notice that in Eq. 8, the constant  $\bar{A}$  includes the stress utilization factor ( $\bar{\sigma}_o / \bar{\sigma}$ ). Thus, one can expect different values of  $\bar{A}$  for axially loaded structures using high strength materials, versus structures that use members in bending or lower strength materials.

To achieve values for constant  $\bar{A}$  we appeal to the proven flight cryogenic support structures detailed in Table 1. Note in Table 1 that  $\bar{A}$  is relatively tightly grouped from around 0.02, for very high efficiency (high stress utilization) structural systems such as 9-band tension systems, and around 0.27 for less-efficient (low stress utilization) cantilever-type structures.

### Example use of the estimation equation

As an example application of Eq.(8), consider a problem of suspending 0.25 kg at 6 K using structural supports attached to 18 K using S-glass support members. This gives us:

$$\begin{aligned} Q \text{ (high stress utilization)} &\approx 0.02 (0.002)(0.25)^{0.66} (12) = 0.2 \text{ mW} \\ Q \text{ (low stress utilization)} &\approx 0.27 (0.002)(0.25)^{0.66} (12) = 2.8 \text{ mW} \end{aligned}$$

where

- Empirical scaling factor  $\bar{A}$  = 0.02 (high stress utilization) to 0.27 (low stress utilization)
- Average conductivity of S-glass between 6 K and 18 K ( $\kappa=0.002$  watts/cm·K)
- Mass of supported assembly ( $m=0.25$  kg)
- Differential temperature across support structure ( $\Delta T = 12$  K)

As a second example application of Eq.(8), consider a problem of suspending 2 kg at 18 K using structural supports attached to 100 K using S-glass support members. This gives us:

$$\begin{aligned} Q \text{ (high stress utilization)} &\approx 0.02 (0.0035)(2)^{0.66} (82) = 9 \text{ mW} \\ Q \text{ (low stress utilization)} &\approx 0.27 (0.0035)(2)^{0.66} (82) = 122 \text{ mW} \end{aligned}$$

where

Empirical scaling factor  $\bar{A} = 0.02$  (high stress utilization) to  $0.27$  (low stress utilization)

Average conductivity of S-glass between 18 K and 100 K ( $\kappa=0.0035$  watts/cm·K)

Mass of supported assembly ( $m=2$  kg)

Differential temperature across support structure ( $\Delta T = 82$  K)

## Acknowledgment

The work described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, through an agreement with the National Aeronautics and Space Administration.

## References

- [1] *Combination Methods for Deriving Structural Design Loads Considering Vibro-Acoustic, etc., Responses*, NASA Preferred Reliability Practice No. PD-ED-1211, National Aeronautics and Space Administration, Washington DC.
- [2] Ross RG Jr and Green K. AIRS Cryocooler System Design and Development. Cryocoolers 9. New York: Plenum; 1997. p. 885-94.
- [3] Petrick SW, Miniature Integrated Camera and Spectrometer (MICAS) detector support design. Jet Propulsion Laboratory. Private Communication.
- [4] Crumb D, Herschel Space Observatory Spectral and Photometric Imaging Receiver (SPIRE) bolometer support cryogenic design. Jet Propulsion Laboratory. Private Communication.
- [5] Collins SA, Rodriguez, JI and Ross, RG Jr. TES Cryocooler System Design and Development. Advances in Cryogenic Engineering; 47B, New York: Amer. Inst. of Physics; 2002. p. 1053-60.
- [6] Cafferty T, Landsat Thematic Mapper cryogenic radiator design. Private Communication.
- [7] Boyle R, High-Energy Solar Spectroscopic Imager (HESSI) cryostat support design. Goddard Space Flight Center. Private Communication.
- [8] Boyle R, Microwave Anisotropy Probe (MAP) telescope structure cryogenic design. Goddard Space Flight Center. Private Communication.